Since we can't do anything about γ, the specific weight of water, we *can* increase period T by increasing buoy mass m and decreasing waterline area A. See the illustrative long-period buoy in Figure P5.33.

5.34 To good approximation, the thermal conductivity k of a gas (see Ref. 8 of Chap. 1) depends only on the density ρ, mean free path ℓ, gas constant R, and absolute temperature T. For air at 20°C and 1 atm, $k \approx 0.026$ W/m·K and $\ell \approx 6.5E-8$ m. Use this information to determine k for hydrogen at 20°C and 1 atm if $\ell \approx 1.2E-7$ m.

Solution: First establish the variables and their dimensions and then form a pi group:

$$\underset{\{ML/\Theta T^3\}}{k} = \text{fcn}(\underset{\{M/L^3\}}{\rho}, \underset{\{L\}}{\ell}, \underset{\{L^2/T^2\Theta\}}{R}, \underset{\{\Theta\}}{T})$$

Thus $n = 5$ and $j = 4$, and we expect $n - j = 5 - 4 = \mathbf{1}$ single pi group, and the result is

$$k/(\rho R^{3/2} T^{1/2} \ell) = \text{a dimensionless constant} = \Pi_1$$

The value of Π_1 is found from the air data, where $\rho = 1.205$ kg/m^3 and R = 287 m^2/s^2·K:

$$\Pi_{1,air} = \frac{0.026}{(1.205)(287)^{3/2}(293)(6.5E\text{–}8)} = 3.99 = \Pi_{1,hydrogen}$$

For hydrogen at 20°C and 1 atm, calculate $\rho = 0.0839$ kg/m^3 with $R = 4124$ m^2/s^2·K. Then

$$\Pi_1 = 3.99 = \frac{k_{hydrogen}}{(0.0839)(4124)^{3/2}(293)^{1/2}(1.2E\text{–}7)}, \quad \textit{solve for } k_{hydrogen} = \mathbf{0.182}\ \frac{\mathbf{W}}{\mathbf{m{\cdot}K}} \quad \textit{Ans.}$$

This is slightly larger than the accepted value for hydrogen of $k \approx 0.178$ W/m·K.

5.35 The torque M required to turn the cone-plate viscometer in Fig. P5.35 depends upon the radius R, rotation rate Ω, fluid viscosity μ, and cone angle θ. Rewrite this relation in dimensionless form. How does the relation simplify if it is known that M is proportional to θ?

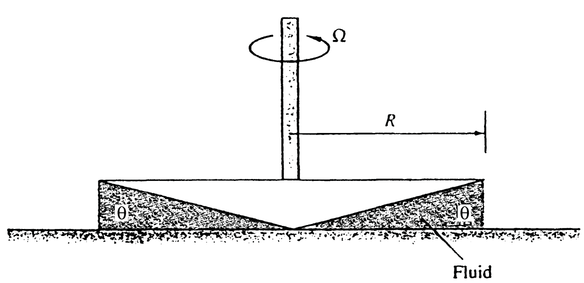

Fig. P5.35

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} M & = \text{fcn}(& R, & \Omega, & \mu, & \theta\ ) \\ \{ML^2/T^2\} & & \{L\} & \{1/T\} & \{M/LT\} & \{1\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect n − j = 5 − 3 = **2** Pi groups, capable of only one reasonable arrangement, as follows:

$$\frac{\mathbf{M}}{\mu\mathbf{\Omega R^3}} = \mathbf{fcn}(\boldsymbol{\theta}); \quad \text{if } M \propto \theta, \quad \text{then } \frac{\mathbf{M}}{\mu\mathbf{\Omega}\boldsymbol{\theta}\mathbf{R^3}} = \mathbf{constant} \quad \textit{Ans.}$$

See Prob. 1.56 of this Manual, for an analytical solution.

5.36 The rate of heat loss, Q_{loss} through a window is a function of the temperature difference ΔT, the surface area A, and the R resistance value of the window (in units of ft^2·hr·°F/Btu): Q_{loss} = fcn(ΔT, A, R). (a) Rewrite in dimensionless form. (b) If the temperature difference doubles, how does the heat loss change?

Solution: First figure out the dimensions of R: {R} = $\{T^3\Theta/M\}$. Then note that n = 4 variables and j = 3 dimensions, hence we expect only 4 − 3 = *one* Pi group, and it is:

$$\Pi_1 = \frac{Q_{loss}R}{A\,\Delta T} = Const, \quad or: \quad \boldsymbol{Q_{loss} = Const\frac{A\,\Delta T}{R}} \quad \textit{Ans. (a)}$$

(b) Clearly (to me), Q ∝ ΔT: **if Δt doubles, Q_{loss} also doubles**. *Ans.* (b)

P5.37 The volume flow Q through an orifice plate is a function of pipe diameter D, pressure drop Δp across the orifice, fluid density ρ and viscosity μ, and orifice diameter d. Using D, ρ, and Δp as repeating variables, express this relationship in dimensionless form.

Solution: There are 6 variables and 3 primary dimensions (MLT), and we already know that

j = 3, because the problem thoughtfully gave the repeating variables. Use the pi theorem to find the three pi's:

$$\Pi_1 = D^a\rho^b\Delta p^c Q \ ; \ \text{Solve for } a = -2,\ b = 1/2,\ c = -1/2. \quad \text{Thus} \quad \Pi_1 = \frac{Q\,\rho^{1/2}}{D^2\,\Delta p^{1/2}}$$

$$\Pi_2 = D^a\rho^b\Delta p^c d \ ; \ \text{Solve for } a = -1\ \ b = 0\ \ c = 0. \quad \text{Thus} \quad \Pi_1 = \frac{d}{D}$$

$$\Pi_3 = D^a\rho^b\Delta p^c \mu \ ; \ \text{Solve for } a = -1,\ b = -1/2,\ c = -1/2. \quad \text{Thus} \quad \Pi_1 = \frac{\mu}{D\,\rho^{1/2}\Delta p^{1/2}}$$